A classical attempt at self-inductance

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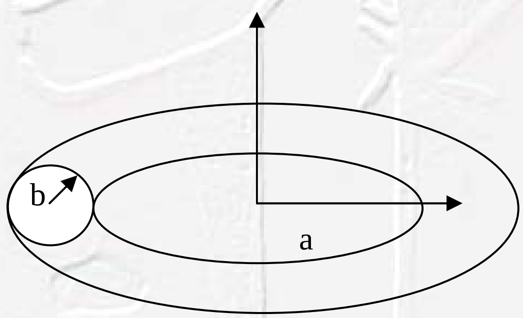
The Self induction equation found in Electrodynamics 3ed by Jackson

In chapter 5 of the book Classical Electrodynamics (3rd Ed) by Jackson (the hot link to purchase the book can be found in the page where this paper was downloaded from), we allegedly find a solution for the self-inductance of a single turn loop. The expression is given as



$$L = \mu_0 a\left[\ln\left(\frac{8a}{b}\right)-\frac{7}{4}\right]$$ for this diagram

In our Anomalies of Classical Electromagnetism paper (apoce.pdf) we state that it is impossible to derive self inductance from Classical Electrodynamics. This position still holds since the above expression is not the self-induction of a loop. We will walk through the derivation of the above equation (*) to show that the expression is actually an approximation for the mutual inductance between two filamentary loops.

Though the attempt was valiant, it is defeated by improper use of vector magnetic potentials.

*** In case you wish to following along we recommend that you purchase the book Classical Electrodynamics 3ed by Jackson. You can find the link on the web page for this paper.**

1 Introduction

This paper will show two separate derivations of the problem 5.32 of page 233 in the book *Classical Electrodynamics* 3rd Edition by John David Jackson. The first will follow that presented by Jackson, the second will be the New Electromagnetism equivalent. Both derivations are done in parallel. Most of you will find that the New Electromagnetism derivation is much simpler and easier to see what is really going on. On the other hand, the classical derivation is obscure and misleading.

We do not know if Jackson is the originator of the derivation for problem 5.32; however, we will call it Jackson's derivation throughout this text for the sake of convenience.

2 Questions

Before we present the derivation of self-inductance found in Jackson, we present some intriguing questions in order to put your mind in the proper frame of thought.

Jackson used magnetic potentials to derive an approximation for the self-inductance of a single turn loop. We offer these questions in the beginning to give you something to think about during the derivation.

The following questions are not intended to "disprove" magnetic potentials.

These questions will be answered in the text that follows.

2.1 What is Magnetic Potential?

Have you every really studied your texts books on this subject? Few actually do.

Magnetic potentials are a mathematical construct developed to give the science of magnetism a tool which is analogous to voltage. There are two types of magnetic potentials: Vector Magnetic Potentials and Scalar Magnetic Potentials. We will provide a brief intro to magnetic potentials in the next chapter.

Because magnetic potentials are only mathematical constructs, they can not be verified by experiment. It is asserted that the Aharonov-Bohm experiment proves the existence of Vector Magnetic Potentials; however, we will publish another paper which shows that the explanation of the Aharonov-Bohm effect is also an improper application of Vector magnetic potentials. In fact, the effect can be explained in terms of the fundamental force equations of New Magnetism.

2.2 Why does it give an answer?

How can the Vector Magnetic Potential (VMP) method give us an answer to the self-induction problem where the fundamental Maxwell equations do not?

The VMP is only an abstraction of Maxwell's equations; therefore, how can it explain more?

If we use electric potential as an example, we see that the fundamental force equation (Coulomb) and the electrostatic Potential derived from it are consistent. In other words, any system modeled by one (E or V) can be worked back into the other. Consequently, if a system is not explainable by one, then it should not be explainable by the other.

How it is possible that Jackson is able to get an answer (for self inductance) from the VMP when it is not possible to obtain one from the fundamental force equations?

Something is wrong! And we will show you.

3 Magnetic Potentials

There are actually two types of magnetic potentials: vector magnetic potentials (VMP) and scalar magnetic potentials (SMP).

The SMP is just a negated Ampere's Circuital Law where the closed loop restriction is removed as such

$V_{M,ab} = -\int_a^b \mathbf{H} \bullet d\mathbf{L}$ this has the units of "amps", but where those amp are is ambiguous (and thus violates the ambiguity rule of Rules of Nature).

The VMP is a more convoluted derivation but ends up as such:

$\mathbf{A} = K_M \oint \frac{Id\mathbf{L}}{R}$ Where $K_M = \frac{\mu}{4\pi}$ and A is flux/meter such that

$$\Phi = \oint \mathbf{A} \bullet d\mathbf{L}$$

Magnetic potentials were developed to provide an analog to Electric potential. With that said, here are some important differences:

	Electric Potential	Magnetic Potential
Units	Measures potential energy per charge (Volts)	It does not represent potential energy.
Measurability	Can be directly measured. Many instruments read out in volts (DVM, Scopes, etc)	Can not be directly measured. No device is made (that we are aware of) which reads out in magnetic potential. There is no experiment which can prove that magnetic potentials exist.
Acts directly on charges	Y	N
Basis for circuit theory	Used in electric circuit theory	Not used in magnetic circuit theory
Vector	Not a vector	There is both a scalar and vector version.

Because VMP is used in the Jackson derivation, we will concentrate our discussion on that.

3.1 The Vector Magnetic Potential

3.1.1 The classical version

In classical electrodynamics the vector magnetic potential is defined by the following:

$$1)\ \mathbf{A} = K_M \oint \frac{Id\mathbf{L}}{R} \quad \text{where } K_M = \frac{\mu}{4\pi}$$

A is only a partial result; therefore, in order to make use of A it must be integrated around a closed loop as such.

$$2)\ \Phi = K_M \oint_S \oint_T \frac{I_S dL_S \bullet dL_T}{r}$$

The above equation only requires a few more operations to become the Neumann equation. In our paper ni_neumann.pdf we show quite exhaustively that classical theory requires that the integration of the vector magnetic potential must be closed otherwise it would be describing the spherical field of New Electromagnetism.

3.1.2 The New Electromagnetic Version

We can derive a New Electromagnetic version of vector magnetic potentials from the New Induction equation

New Induction

$$1)\ emf = -K_M \int_S \int_T \left(\frac{dI_S}{dt}\right) \frac{dL_S \bullet dL_T}{|\mathbf{r}|}$$

Then drop the dLT

$$2)\ E_M = -K_M \int_S \left(\frac{dI_S}{dt}\right) \frac{dL_S}{|\mathbf{r}|}$$

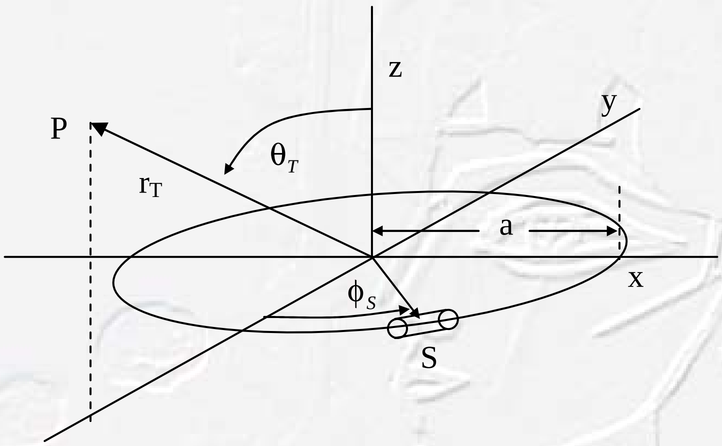
Integrate both sides by time and multiply by -1 to yield

3) $-\int E_M\,dt = K_M \int_S \frac{I dL_S}{|\mathbf{r}|}$

The right hand side of the equation is now identical to the classical definition of the vector magnetic potential except that it is not constrained to closed loops. You will notice that the left hand side is just as obscure as the classical definition for vector magnetic potential. Therefore, we will use the equation in step 2 for the New Electromagnetic magnetic vector potential except that we will instead call it the Magnetic Electric field (non conservative)

$$E_M = -K_M \int_S \left(\frac{dI_S}{dt}\right) \frac{dL_S}{|\mathbf{r}|}$$

4 The Induction Problem



The above diagram is similar to the one used by Jackson on page 182 except that we are using New Electromagnetism notation (see ne.pdf for detailed list) which is less confusing.

The induction problem requires a number of steps before a value for induction is actually derived.

4.1 The first step

In the first step we are required on page 182 to find the Vector Magnetic potential at point P about a "Filamentary" current source.

Jackson arrives at the following in equation 5.36 on page 182

J1) $$A\phi(r,\theta) = K_M I_S a \int_0^{2\pi} \frac{\cos\phi_S \, d\phi_S}{\sqrt{a^2 + r_T^2 - 2ar_T \sin\theta_T \cos\phi_S}}$$ (Jackson (5.36))

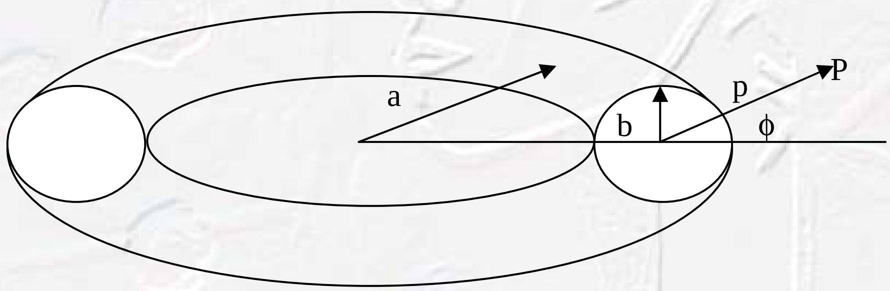
Applying the New Electromagnetic version found in section 3.1.2

NE1) $$E\phi(r,\theta) = -K_M \frac{dI_S}{dt} a \int_0^{2\pi} \frac{\cos\phi_S \, d\phi_S}{\sqrt{a^2 + r_T^2 - 2ar_T \sin\theta_T \cos\phi_S}}$$

The above integrations are solved with elliptic integrals to provide decent Approximations. For more information on elliptic integral, see the CRC Standard Math Tables which can be purchased through one of the links found on the page where this paper is downloaded.

4.2 The Second Step

In the second step, we are to determine the vector magnetic potential at a point near the surface of a ring using the "Filamentary" approximation for vector magnetic potential from the first step.



In the above a is much much larger than b.

Jackson arrives at

J2) $A_\phi = 2K_M I_S \left[\ln\left(\frac{8a}{p}\right) - 2 \right]$ (Jackson (problem 5.32a))

Applying The New Electromagnetic version

NE2) $E_\phi = -2K_M \frac{dI_S}{dt} \left[\ln\left(\frac{8a}{p}\right) - 2 \right]$

I ask you, which equation gives you a clearer picture of what is really going one here? The Vector Magnetic Potential which really makes no sense or the magnetic electric field.

The New Electromagnetism equation states that there is a force-per-charge (E field) at point P that kicks charges in the opposite direction to the current change at the center of the wire.

At this point we will continue with Jackson and get back to New Electromagnetism later.

4.3 Jackson 3rd Step (5.32b)

In the third step Jackson requires us to determine an expression for the vector magnetic potential at a point inside of the wire. The purpose of this step is to account for intrinsic inductance of the wire.

He assumes that the vector magnetic potential in J2 is correct for points outside a wire regardless of the distribution of current inside the wire. He then uses J2 as a boundary condition for the magnetic potential for points inside a conductor carrying a uniform current distribution.

I believe this is where the author becomes confused between Scalar Magnetic Potential (SMP) and Vector Magnetic Potentials (VMP).

In Scalar Magnetic Potentials (SMP), the above assumption is correct because SMP is based on Ampere's Circuital Law. In SMP, the closed path integral of H around a wire always reflects the amount of current passing through the path regardless of the distribution of current.

If this were true in Jackson's assumption then the VMP multiplied by the length of a circle of radius p would equal a constant ($A_\phi 2\pi p = const$) and this is not true.

In any event, the result obtained by Jackson is

J3) $A_\phi = K_M I_S \left[1 - \frac{p^2}{b^2}\right] + 2K_M I_S \left[\ln\left(\frac{8a}{b}\right) - 2\right]$ for p<b

Although this step is erroneous, it affects the result by very little as you will see.

4.4 Jackson 4rth step (5.32c)

In the fourth step we are required to use equation 5.149 in order to solve for the magnetic energy and hence the self inductance.

Equation 5.149 is $W = \frac{1}{2}\int \mathbf{J} \bullet \mathbf{A} d^3 x$

And the result Jackson gets is

J4) $L = K_M 4\pi a\left[\ln\left(\frac{8a}{b}\right) - \frac{7}{4}\right]$

Notice that the intrinsic inductance accounted for in step 3 only changed the 2 to 7/4. This is a minor change and has little effect on the overall inductance.

4.5 New Electromagnetism Solution

The New Electromagnetism solution is much more straightforward that the convoluted method found in Jackson's book.

We simply observe that a forward accelerating current in the ring will generate a "Backward electric field". Thus any forward moving current will pass where the back emf is least (path of least effort). This occurs at the outside of the wire. Since emf is E*distance and E is substantially uniform around the ring, then simply multiply NE2 by the perimeter of the ring and set p to b; thus:

NE3) $emf = -K_M 4\pi a \frac{dI_S}{dt}\left[\ln\left(\frac{8a}{b}\right) - 2\right]$

Then we remember that $L = -\frac{emf}{\frac{di}{dt}}$

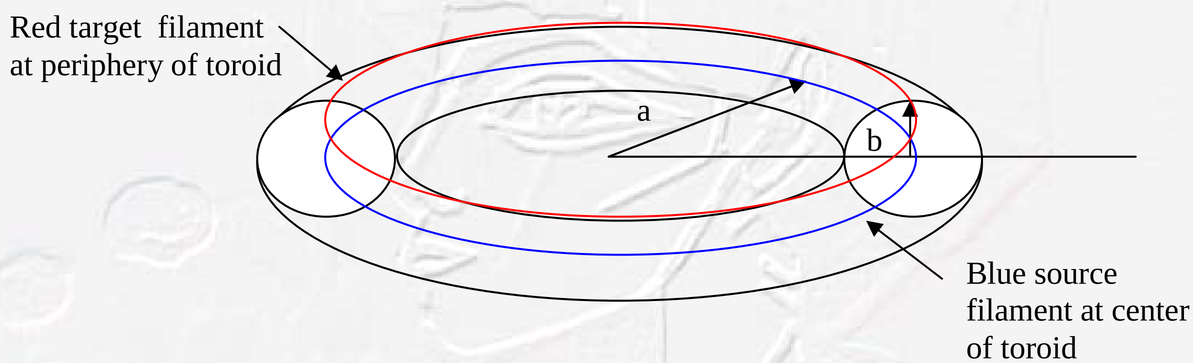
Thus

NE4) $L = K_M 4\pi a\left[\ln\left(\frac{8a}{b}\right) - 2\right]$

Of course, this does not account for intrinsic inductance as Jackson does; however, this point is moot since neither of these equations are truly the inductance of a single turn loop.

5 What did we really do?

We went on a long drawn out adventure to calculate the mutual inductance between a filamentary current ring at the center, and another located at the edge. This is shown more clearly by the following diagram.



Therefore, we did not solve for self inductance; instead, we developed an approximation by calculating the mutual inductance between two loops.

You can see that the New Induction approach was vastly simpler and easier to follow. It also represented more clearly what was happening.

6 Conclusions

There are multiple conclusions that we can draw from this exercise. We have a subchapter for each conclusion.

6.1 This is not self-Inductance

It should be clear to the reader that the values for inductance derived in the methods presented in this paper are not true representations of the self-inductance of a single turn loop of wire. The equations are in fact approximations for the mutual inductance between two filamentary loops of wire.

In section 6.3 we will explain how New Electromagnetism solves the self induction problem.

For a "two-loop" approximation, the results obtained are not too bad. The following table compares Jackson to experimental results:

48 inch perimeter loops	Measured inductance nH	Jackson nH
22 AWG wire	2055	1611
26 AWG wire	2655	1724

With more experiment, it might be possible to develop a correction factor that could adjust Jackson to experimental results. We leave this exercise to the motivated reader.

6.2 Vector Magnetic Potentials

As stated in the opening section of this document, Vector Magnetic Potentials are confusing. Not only are they contradictory in definition, they actually hide the true essence of what is being derived (as demonstrated in this paper).

In fact, the derivation found in Jackson should have set off alarm bells years ago. How can vector magnetic potentials describe more than the basic models from which it was derived? How come nobody questioned this blatant contradiction? To say it another way: if it is not possible to derive a relationship for self-induction from the fundamental electromagnetic force equations (Maxwell, Faraday, et al), then it can not be possible to do it from an abstraction of those same models (such as Vector Magnetic Potentials).

The paradox is resolved by realizing that the results of the derivations are not self inductance; instead, we did an awful lot of math to solve a stupid mutual inductance problem.

In any event, New Electromagnetism does away with Vector Magnetic Potentials. There are no applications which traditionally required Vector Magnetic potentials that can not be solved with the New Electromagnetic fundamental force equations. As demonstrated in this paper, the New Induction solution was simpler and easier to follow. It enabled us to see more clearly what was being derived.

False Advertising

It is claimed that the Jackson's derivation is for a loop with a uniform current density in the cross section. If this is so, then why does the derivation hinge upon a filamentary current distribution localized at the core of the wire?

6.3 What is the solution

The actual solution to self-induction is much more complicated than the attempt presented in Jackson. Simply applying New Induction to a filamentary loop will not do either. The problem requires breaking a wire down in to finite volumes. Each volume contains a certain number of free and excess carriers (See New Magnetism) whose motions (velocity, acceleration and concentration) all must be accounted for.

Such a solution can only be handled by computer and requires intense computer time. In order to reduce computer time without loss in accuracy, we have developed proprietary finite element modeling algorithms called Fractal Finite Element Algorithms that can reduce the computation time of such complex interactions by 3 orders of magnitude (1000).

Though the details of the algorithms will be protected, they will be used in upcoming software products and books. One such book is NIA2 (New Induction Applications volume 2) which concentrates on self-induction, intrinsic induction, skin effect and related topics. It will include a software package based on the Fractal algorithms to enable the reader to verify the capabilities of the new algorithms.

The book New Magnetism will also feature new software releases for modeling magnetic fields in motion and enable the user to calculated real world interactions.

7 Document history

1.0) Initial Release
1.1) Typos corrected; clarified section 2.1

