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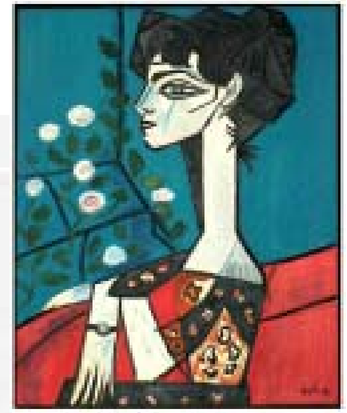
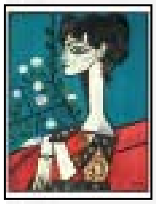


Level 5 Test Answer



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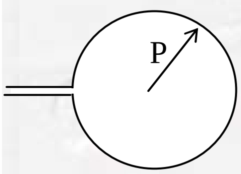
Level 5 Test Answer

ABSTRACT

The following problem, in electromagnetic theory, is asked of visitors to our website in order to help them determine whether they qualify as a skill level 5 (expert) in classical electromagnetic theory.

The problem is stated as follows:

Derive an expression for the self-inductance of an air-core, single-turn, circular loop inductor of radius P using **classical electromagnetic field models**.



Single turn loop of radius P

Note 1: The inductance of an inductor is comprised of two components. The first is the self-inductance and the second is the intrinsic inductance (some call it the internal inductance). All that is required for this test is the self-inductance component of inductance.

Note 2: You should know that Faraday's Law and the Biot-Savart law (a.k.a. Ampere's Law) are the two models necessary to derive the expression. If you think that it would be easier with Maxwell's equations, then you should review the derivation of Maxwell's Equations.

Note 3: There may be software packages out there that can give you a solution to this problem using empirical methods; however, we are testing the classical field models and your ability to apply them. So don't cheat. It is OK to use math tools (math cad, calculator, integration tables, etc). Most classical Electromagnetic texts show how to apply Faraday's Law to the above problem; however, none actually provide a solution (if you find one let us know).

Note4: There is an expression found in classical electrodynamics text books that obtains $L = \mu a\left[\ln\left(\frac{8a}{b}\right) - \frac{7}{4}\right]$; however, this expression is not correct. See **http://www.distinti.com/publications/ind_jackson.htm** for more details.

This problem can not be solved with classical electromagnetic theory. Those people who have not actually tried to solve this problem may find this hard to believe.

The solution

Those readers who actually took the time to attempt the problem may have run into a division-by-zero condition which prevented them from obtaining a sensible answer. Some readers may have resorted to extreme mathematical methods in an attempt to force an answer; however, excessive techniques should not be required for such a simple problem. For example, if we compare inductors to other devices whose operation is directly described by the behavior of electromagnetic fields ("field devices") we find that inductors are an exception (see the following table).

Commonly used field devices	Applicable field model(s)	Results when models are applied
Inductors	Faraday's Law	**Undefined results using standard methods**
Capacitors	Coulomb's Law	Very accurate results using standard methods
Transformers	Faraday's Law	Very accurate results using standard methods
Motors	Motional Electric Law	Very accurate results using standard methods
Generators	Faraday's Law or Motional Electric Law	Very accurate results using standard methods

It is clear from the above table that, except for inductors, all other field devices are accurately modeled using classical theory and standard methods. Why must we resort to non-standard methods in order to get a sensible answer for an inductor?

When I came across the inductor problem many years ago, I spent many days trying different techniques and tricks in order to obtain a sensible general-case solution. I tried limiting the lower integration limit; none of the "sensible" lower limits proved useful. I tried normalization techniques which were also fruitless.

After days of frustration, the oddness of the whole situation congealed in my thoughts. I reasoned that if an inductor is one of the most basic components of electric circuit theory AND if all of the other basic components (in the table above) yield accurate answers using standard methods, then why should inductors be different? Perhaps it is only the definition of inductance that makes Faraday's Law difficult to apply? This is not likely since inductance from the point of view of circuit analysis is beautifully symmetric with other 2nd order analogs (such as mass in a second order mechanical system).

Perhaps if we study the other component of inductance (intrinsic inductance) we might be able to understand the phenomenon of self-inductance better. According to classical texts, intrinsic inductance is not derived using Faraday's Law (???). Instead, the common derivation uses the amount of field energy contained in the wire and "reverse-engineers" the expression for inductance. According to the derivation, intrinsic inductance is a linear function of wire length and independent of wire diameter. Thus, according to the classical understanding of inductance, if we construct two circular loops of wire, both with the same loop diameter, but with different wire gauge, then both should have the same inductance. But this is not the case.

We have come to a curios dilemma; neither of the components of inductance (self inductance and intrinsic inductance) can be determined using standard methods.

If we are forced to use non-standard methods, then why not see if there is another mathematical relationship for induction (there is no law that says that there is only one mathematical relationship for any given physical phenomenon). To see what we should expect from another relationship, we study the general form of the classical models using the following table:

Classical field Model	Type	Applicable to point charges
Faraday's Law	**Scalar**	**NO**
Coulomb's Law	Vector	YES
Motional Electric	Vector	YES

Logically then, there should exists a vector point equation which relates charge acceleration to force. This discussion is continued in the paper "New Induction" which is published for free (download only) at our website. For those who didn't quite understand the preceding discussion:

The answer: the problem can not be solved using classical models.

The ultimate irony is that classical electromagnetism textbooks (at least the ones that I have seen) use Faraday's Law from a conceptual standpoint to explain the theory of inductors; however, not one actually shows a solution; undoubtedly because it can not be done.

There is a classical electrodynamics textbook that obtains an answer using vector magnetic potentials (see note 4 on first page); however, we show quite conclusively that this result is erroneous. In fact the result is nothing more than an approximation for the mutual inductance between two filamentary loops. For more info, see the link found in note 4 on first page.

Document History

1.0) Initial release (1999)

1.1) added note 4 regarding Jackson